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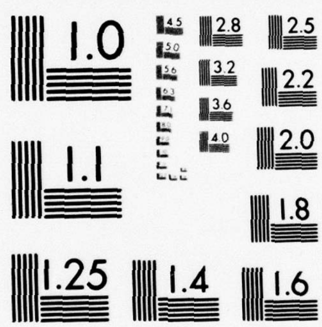

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This thesis studies ~~A~~ constellation of satellites that gives multiple continuous coverage of the whole earth ~~is studied here~~. A computer program is described which helps to find the optimal configuration. In particular, performance of a twenty-four satellite constellation is analyzed for triple coverage and the results given. Constellations where the number of orbital planes equal the number of satellites (N=P) and constellations where N ≠ P are both analyzed. Circular orbits are assumed. The computer program is

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SATELLITE NETWORKS FOR MULTIPLE COVERAGE

by

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B. E. M. E., VILLANOVA UNIVERSITY
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SATELLITE NETWORKS FOR MULTIPLE COVERAGE

by

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ABSTRACT

A constellation of satellites that gives multiple continuous coverage of the whole earth is studied here. A computer program is described which helps to find the optimal configuration. In particular, performance of a twenty-four satellite constellation is analyzed for triple coverage and the results given. Constellations where the number of orbital planes equal the number of satellites (N = P) and constellations where N ≠ P are both analyzed. Circular orbits are assumed. The computer program is relatively short using a sampling grid over a quarter of the globe. The results are approximately correct sufficient for a feasibility study.

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J. G. Walker's work was the most useful reference for the pages that follow, so my thanks extend across the Atlantic for Farnborough, England as well as to Dr. Paul Cefola who dropped the Walker reports on my desk.

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Table of Contents

Introduction

In order to update a vehicle's state vector, single, double, triple or quadruple coverage may be desired from a constellation of artificial satellites. One important consideration to the constellation designers is the optimal configuration in terms of pure geometrical distribution. Thus, what is the best coverage N satellites can produce and what are the Keplerian elements of each satellite at some point in time?

In the literature one can find the optimal coverage for constellations of up to 15 satellites[1]. There is presently a need for better coverage than is afforded by these. The earth central angle between user and satellite is too large for many applications.

J. G. Walker has laid down much of the ground work for analysis of such constellations. What is needed, however, is a method, employing a computer program, to simplify and make possible analysis of systems where N is considerably greater than 15.

Constellations where each satellite is placed in a single orbital plane ($N = P$) and those where equal numbers of satellites are contained in fewer planes ($N > P$) are both considered here. For the $N = P$ case, type I, the inclination is assumed the same for all orbits, the spacing in

longitude of ascending node (LAN) is made equal to 360°/N and the phase angle between satellites is given by F x 360/N. F is an integer whose optimal value, along with inclination, is to be found.

For the N > P case, type II, all possibilities where P is a factor of N are considered. The increment of LAN is then 360/P, the phase angle is defined the same way as for type I constellations, and the inclinations for all P planes is the same.

The 'Grader' and 'Grader 1' programs analyze these two types of constellations respectively. They take a grid of samples over half of the northern hemisphere to give an approximation of the performance of the different possible constellations. It is meant to be an aid to the systems engineer who wants to get an idea of what the constellation will look like. One thus arrives at the most probable optimal configuration, the number of orbital plane (P), the phase angle number (F), and the largest earth central angle (R_{max}) between the user and the satellite which must be tolerated to get the coverage specified.

Section I

A Look at Whole Earth Continuous Coverage

In 'whole earth continuous coverage' one is striving to have at any point in time, anywhere on the globe, a prescribed number of satellites 'visible'. The 'prescribed number' depends of course on the user's intent. If one desires single coverage then at least one satellite should be visible, and so on. Visibility requirements can usually be very neatly defined by a maximum allowable earth central angle between the user and the satellite. In the example chosen in this paper, we are looking for triple coverage with a maximum earth central angle of approximately 45 degrees.

Note that although the coverage requirement may be fulfilled, the type of coverage the satellites afford could be poor for navigation. A measure of the navigability of a given constellation is not included here but is being studied at this time[1].

Results of the study are shown in Table 1. The N/P/F notation denotes the number of satellites (N), the number of orbital planes (P), and the phase angle number (F). R_{max} represents the largest angular distance to acquire three satellites. This number should be as low as possible. The inclination (δ) is to a reference plane which can be, but does not have to be, the equatorial plane. D_{min} represents

Table 1

Summary of Triple Coverage Analysis

Constellation Code N/P/F	Minimum R_{max} (deg)	At Inclination δ (deg)	Minimum Distance Between Any Two Satellites (D_{min}) (deg)
24/24/14	51	53.0	17.1
24/3/1	50	53.3	12.2

the minimum angular distance between two satellites encountered with this scheme. Constellations where D_{min} was equal to zero were discarded due to the relatively high probability of collision for circular orbits of equal altitudes. Poor navigation also results at those instances and places where this occurs. Walker also takes this approach. The assumption is made that the proper station keeping is available to keep the relative geometry constant. Similar perturbations to all the orbits may be ignored such as those shifting the longitudes of ascending nodes.

Section II

Finding the Optimal Configuration

Finding the optimal configuration can be long and tedious if attempted by purely deterministic approaches, especially when the number of satellites is large. There are too many variables involved as can be supported by anyone who has attempted it[2]. In order to speed up the process, a program called 'GRADER' was introduced (see Section IV and Appendix A for more details).

The program narrows the field of contenders to one or two configurations for an N-satellite constellation. It helps to pinpoint the number of orbital planes, F number, and inclination of the orbits. This is accomplished by an iterative method on the part of the user. It is not intended to give an extremely accurate optimal inclination or R_{max}. D_{min}, the minimum distance between two satellites, must also be found by other means. An algorithm which can be used in finding the actual value of the optimal inclination and the corresponding R_{max} are outlined in Section III.

The way the figure of merit is given to a configuration by the Grader program will be discussed in detail in Section IV and Appendix A. Basically the program looks at a sufficiently large sampling area, checking for single, double or triple coverage at each sampling point. The number of failures in

coverage is divided by the number of samples taken to get failures per samples (FPS). To find, then, the best performance possible from a N-satellite constellation, the procedure shown below has been found helpful:

a) One starts with N = P constellations

b) then runs the program for the different possible F numbers at elapsed angle equal to 0. The threshold angle for visibility must be set so that the curves, as shown in Figures 1, 2, 3 to 7, do not bottom off. Inclinations can be set initially to run from 40 to 70 at increments of 5 degrees.

c) The N > P constellations can then be checked by running the program for different values of P. At each P, the meaningful F numbers are checked also at elapsed angle set to zero and inclination varying from 40 to 70.

d) The configurations with the lowest minima can then be checked at other elapsed angles until the inclination with the lowest maximum value is clearly defined.

For N = 24 with triple coverage, the configuration which behaves the best seems to be 24/3/1. This is found from figures 1 to 7 which show that although a few configurations have very low FPS values at E.A. equal to zero, e.g. 24/6/5, 24/12/1, 24/24/8, 24/24/14, 24/24/16, at other E.A. values they have higher minimums than 24/3/1.

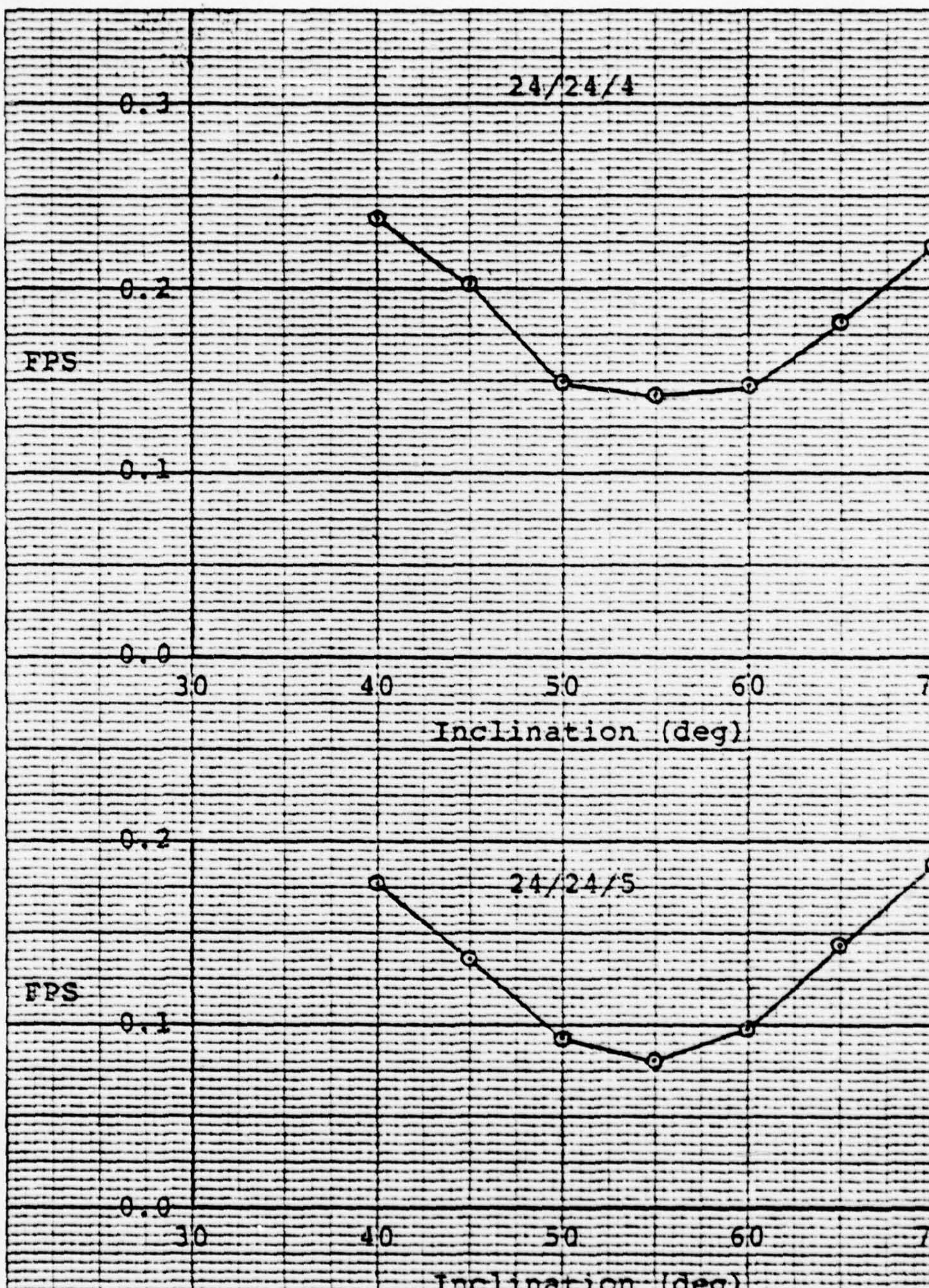

Fig.1 - Failures Per Sample vs. Inclination for different phase angle numbers at Elapsed Angle of zero. The threshold angle (alpha) is set to 45°.

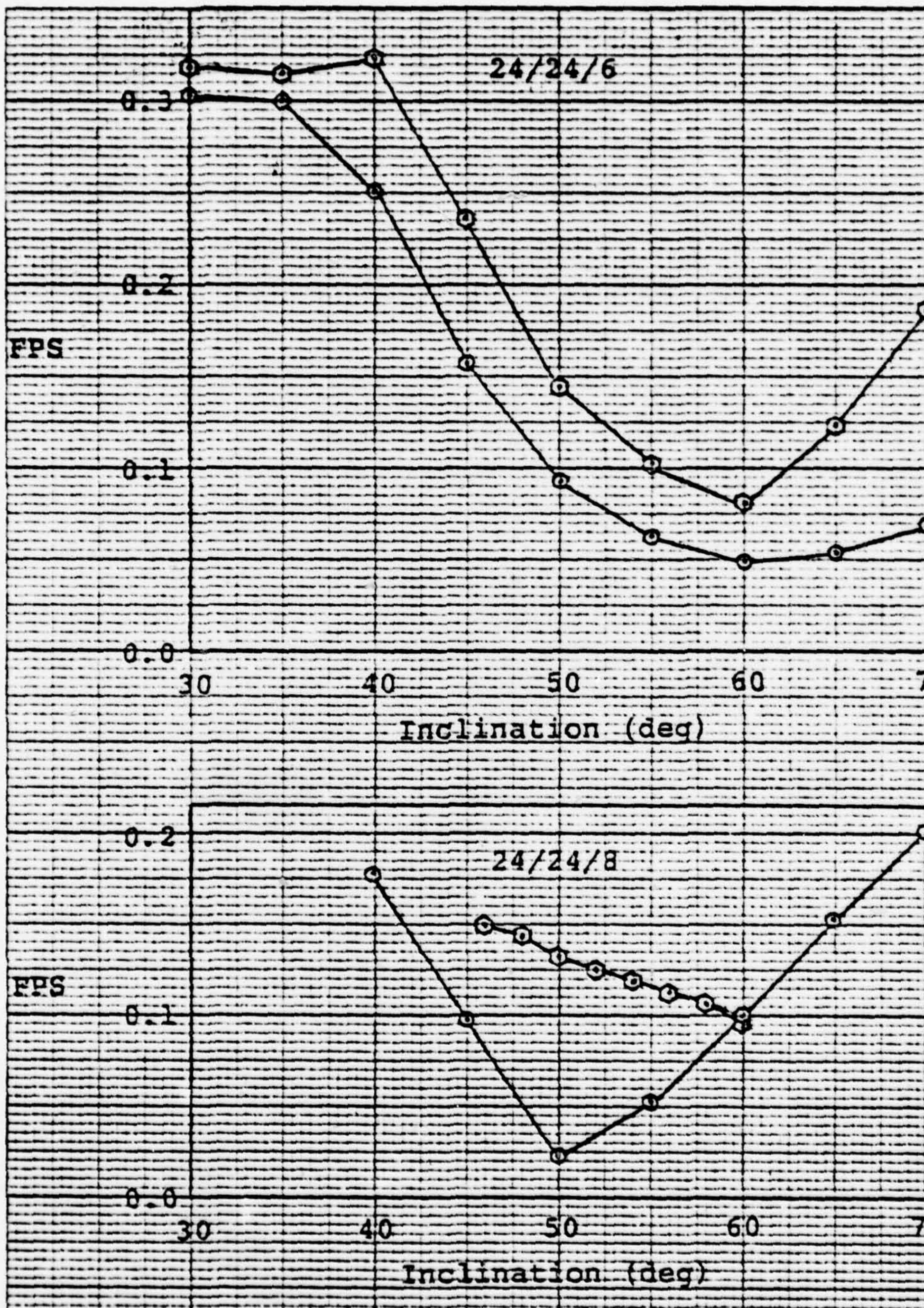

Fig.2 - FPS vs. Inclination for different F nos.

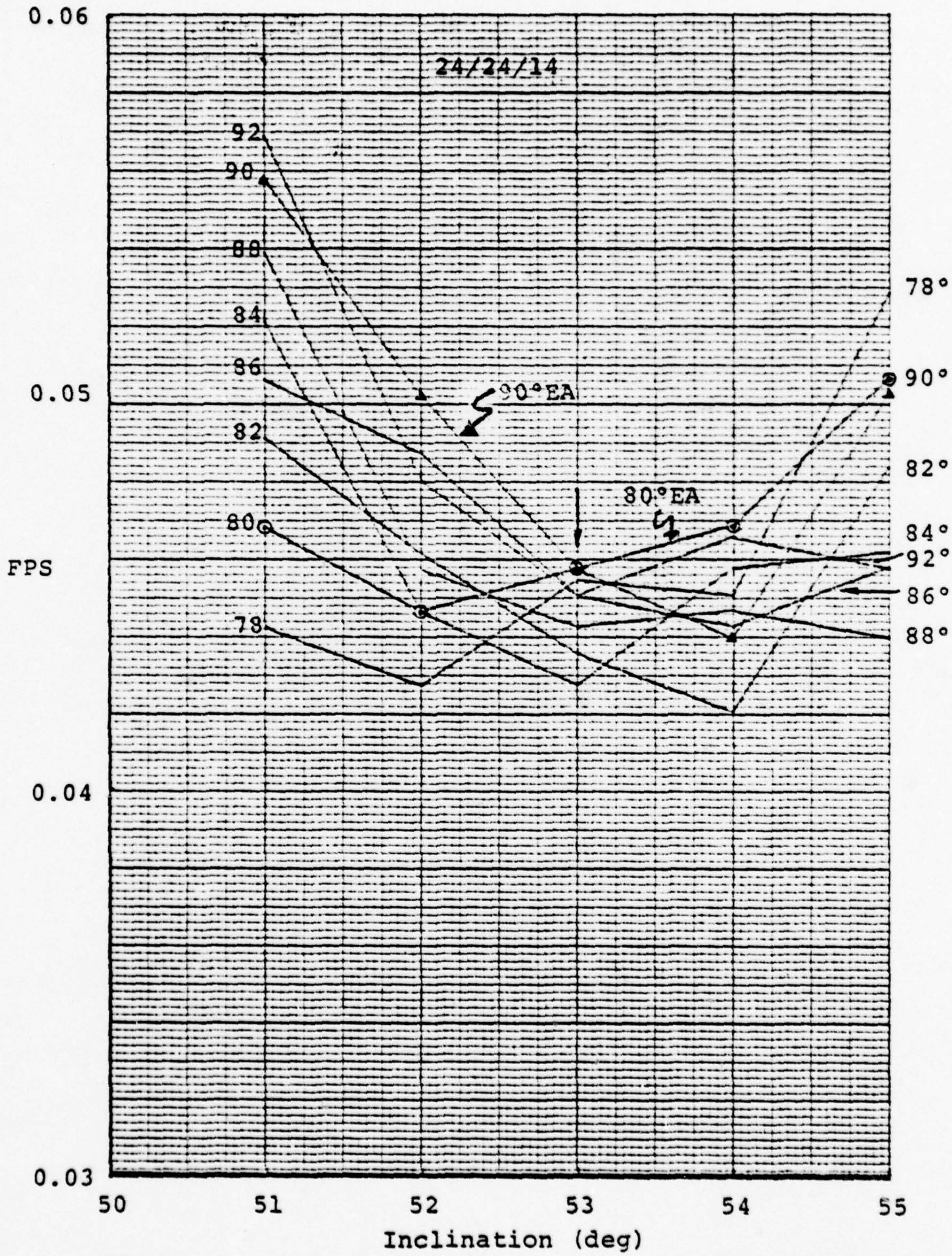

Figure 3 - FPS vs. inclination for constellation 24/24/14

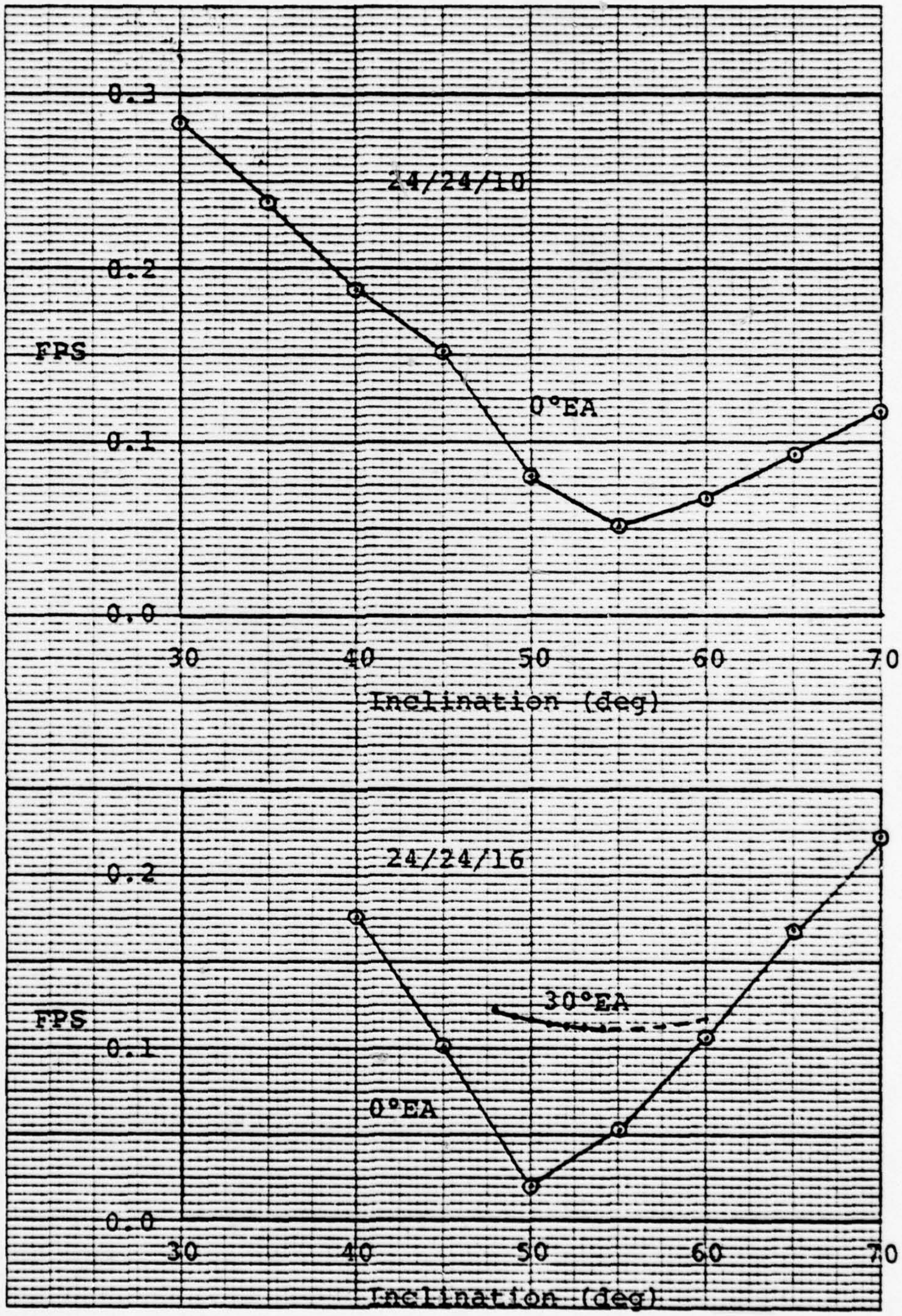

Fig.4 - FPS vs. Inclination for different F nos.

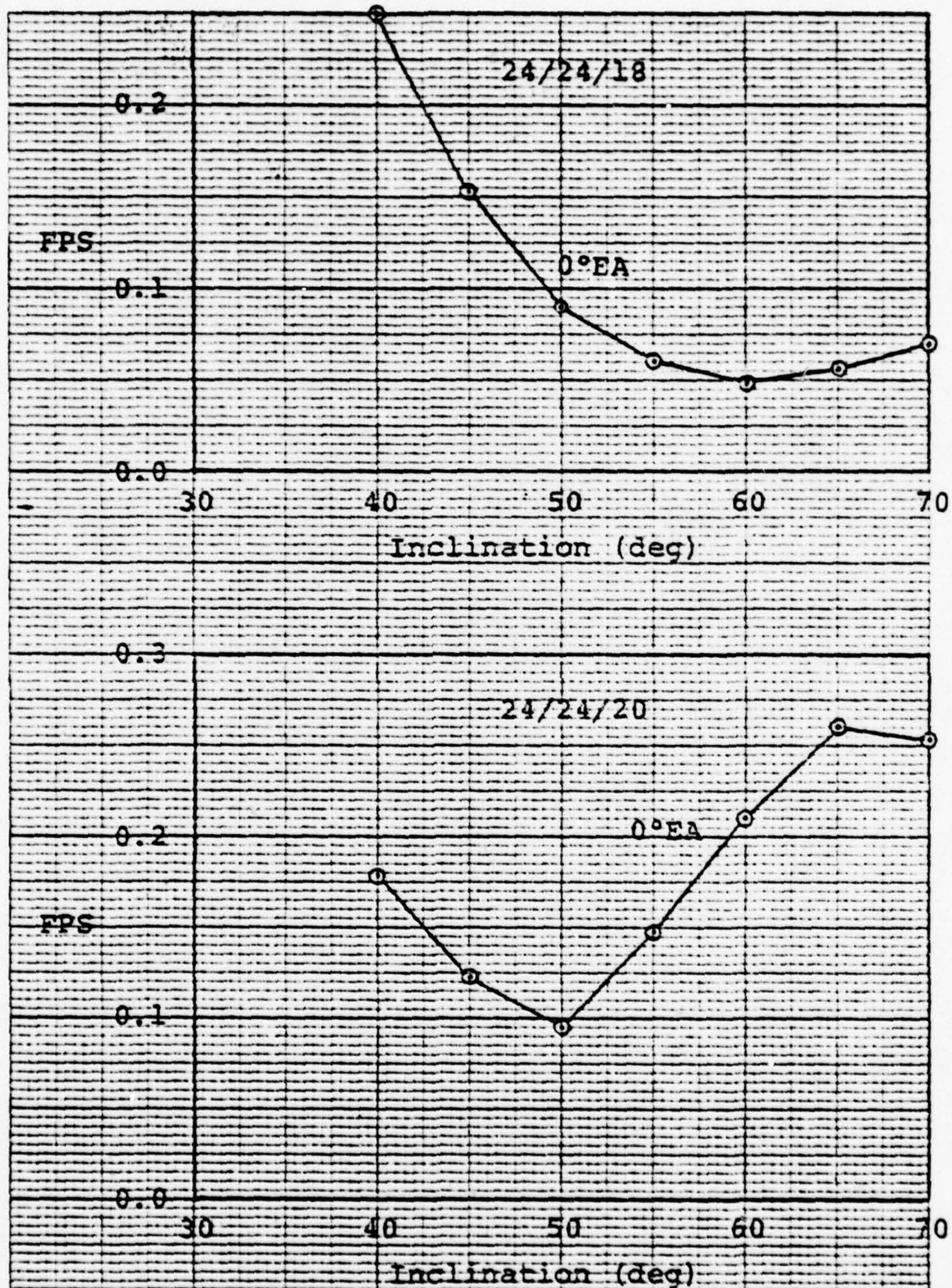

Fig.5 - FPS vs. Inclination for diggerent F nos.

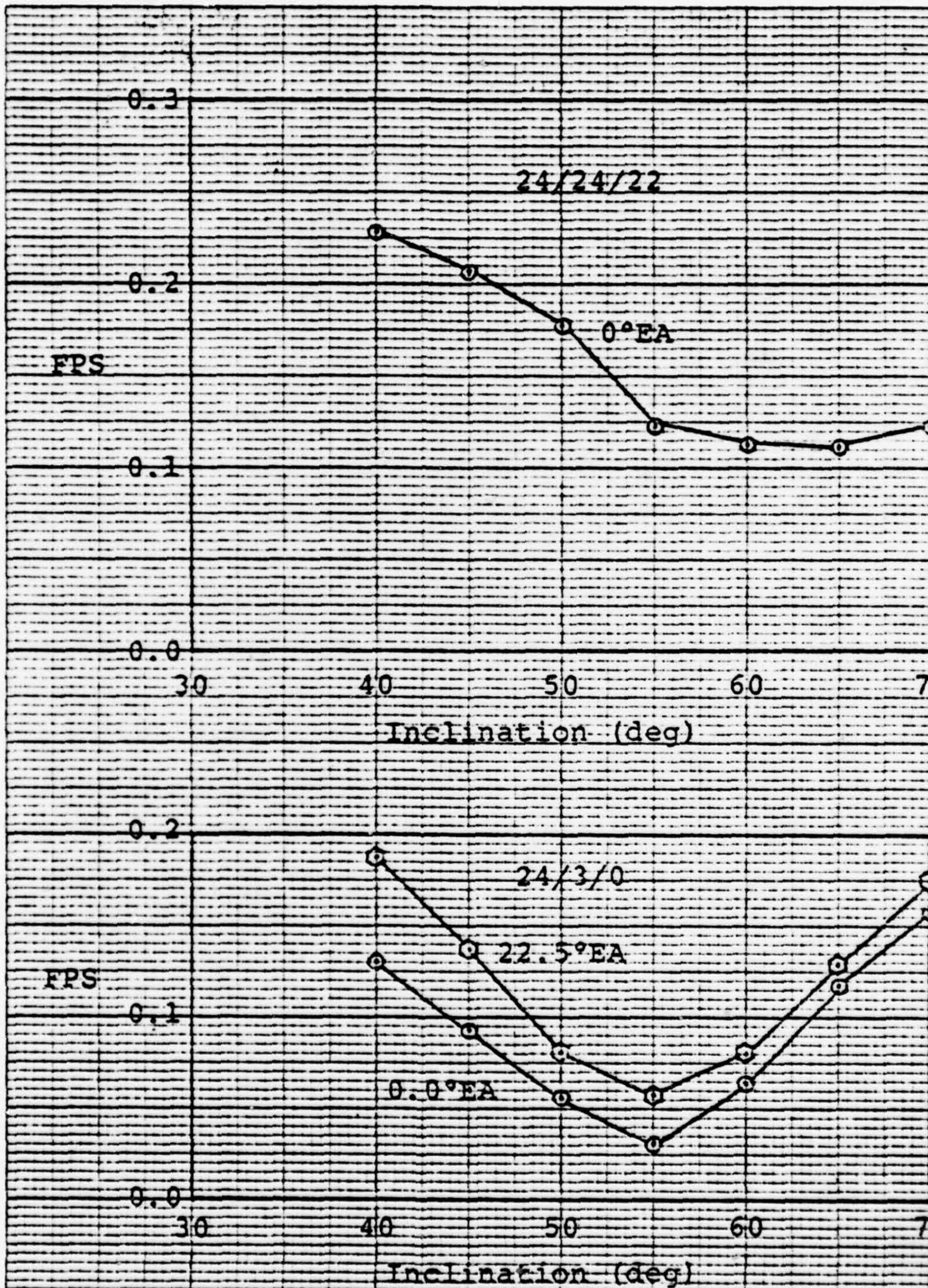

Fig. 5A- FPS vs. Inclination for different F nos. at relevant elapsed angles (EA).

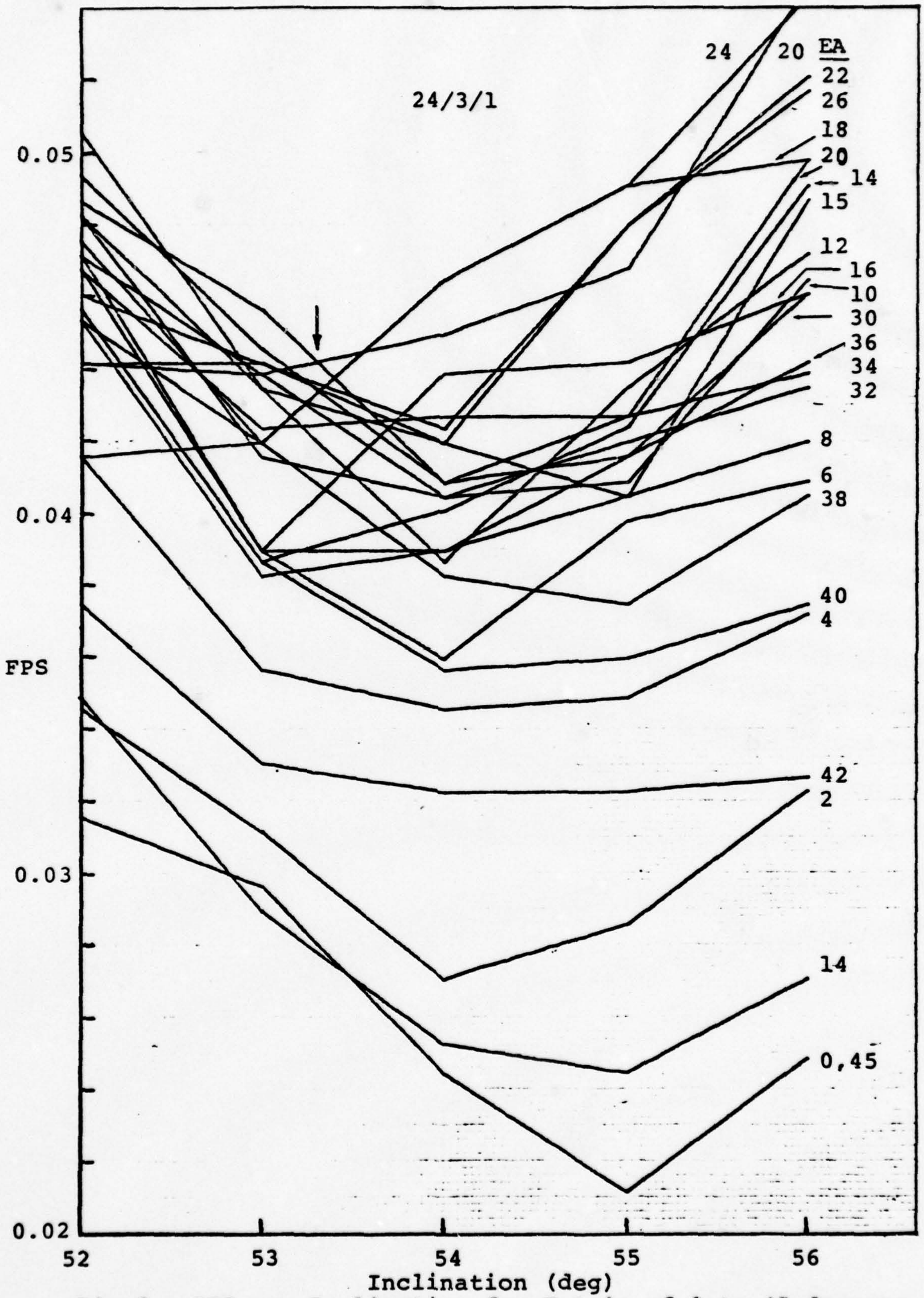

Fig.6 - FPS vs. Inclination for E.A.'s of 0 to 45 degrees.

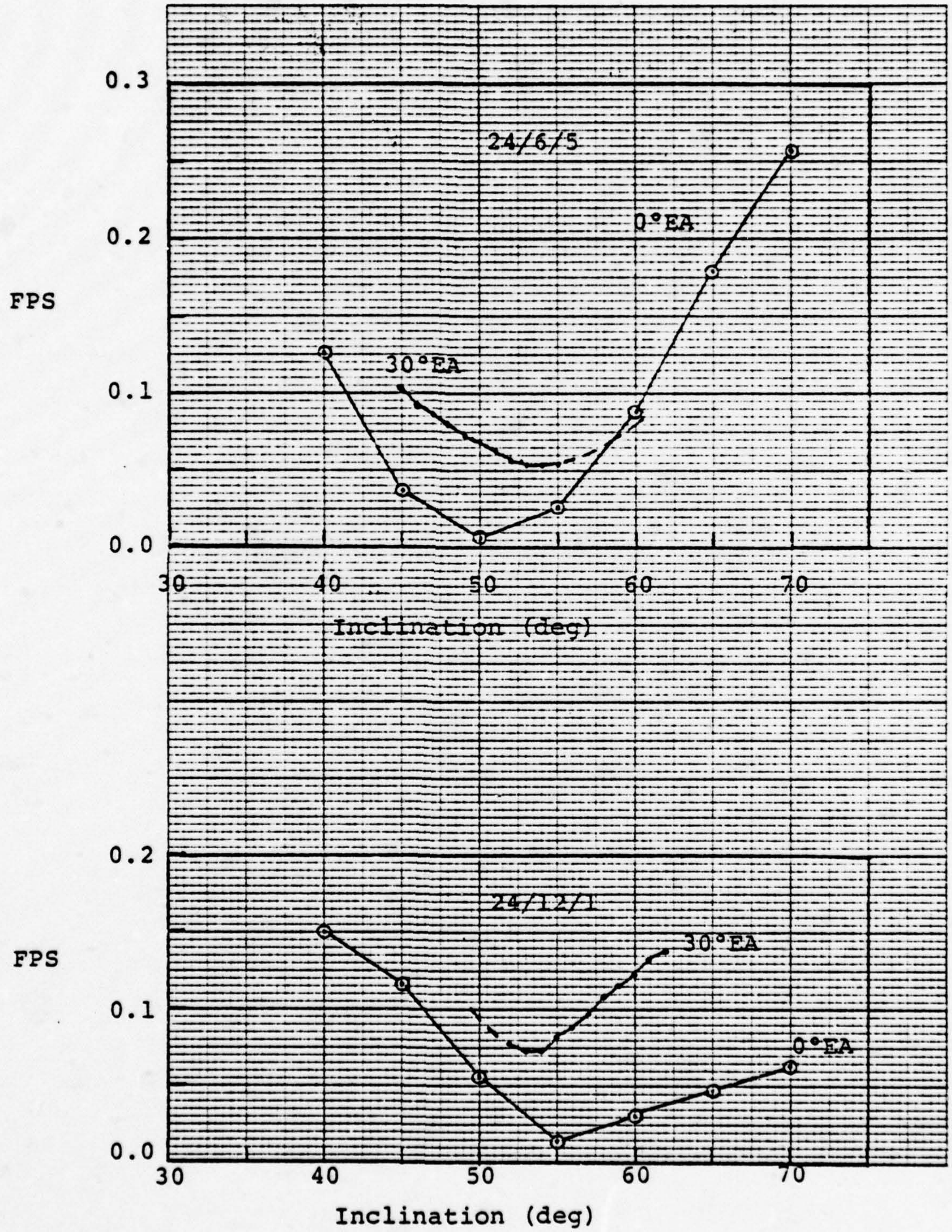

Figure 7 - FPS vs. Inclination.

Three points regarding time saving measures in the search can be made at this point. First, one does not need to vary the elapsed angle until all the F numbers have been analyzed at an EA equal to zero. Only those F values which look promising need to be investigated further. If a few values of F show promise, then the node angle through which the satellites must travel before the geometry repeats itself can be a good guide for the next move in the search. The smaller the repetition angle, the better. An idea of the repetition angle can be had by looking at just the nodes of each satellite and seeing when a satellite will move to the equatorial position, etc. Looking at 24/24/14, we see that with a phase angle of:

$$\begin{aligned}\Delta\omega &= F \times 360/N \\ &= 14 \times 360/24 \\ &= 210 \text{ deg,}\end{aligned}$$

the longitude of ascending nodes and node angles are:

Sat No.	Ω	ω	Sat No.	Ω	ω
0	0	0	6	90	180
1	15	210	7	105	30
2	30	60	8	120	240
3	45	270	9	135	90
4	60	120	10	150	300
5	75	330	11	165	150

Sat. No.	Ω	ω	Sat. No.	Ω	ω
12	180	0	18	270	180
13	195	210	19	285	30
14	210	60	20	300	240
15	225	270	21	315	90
16	240	120	22	330	300
17	255	330	23	345	150

Observing the node angles of satellites 5 and 17, at 330°, as well as 10 and 22 at 300°, and 7 and 19 at 30°, one can be fairly certain that the overall geometry will repeat itself every 30 degrees.

Second, as mentioned before, some phase angle numbers cause satellites to coincide periodically. The odd numbered F's in the 24/24/F constellations were found to have this characteristic. The F's versus inclination graph for 24/24/5 is shown in Figure 1. The indication is that of high failure rates as would be expected.

Thirdly, some F numbers are unlikely candidates for optimality judging by the inherently poor distribution which they offer. For example 24/24/0 means that all satellites will have the same node angle, thus they will all be at the equator initially and move upwards at the same rate. This is obviously poor for global coverage and need not be ana-

lyzed further. F numbers such as 1 and 23 can also be envisioned as rather poor but may still be checked quickly for completeness.

Up to this point only integer numbers have been used for F. Justification for this can be found in references 2 and 3. They have found that fractional values of F can be discarded. One may note that if a fractional value were used, a certain discontinuity would result in the distribution.

The different configurations were initially checked from 40 to 70 degrees in inclination at increments of 5 degrees. This, as opposed to finer plotting, gives a fairly good idea of their performance while saving much computer time. Inclinations below 40 degrees or above 70 were not checked due to the indications of the curves as well as references 3 and 4.

Figures 3 and 6 show the results found from constellations 24/24/14 and 24/3/1 which were the two most promising ones. Note that for 24/24/14, relative to the sampling grid, there is repetition after 360 degrees of elapsed angle at which time the satellite numbered 0 is back at the equator. Therefore, every 10 degrees of elapsed angle was checked up to 360 and increments of 2 degrees were used in the area of interest. Only elapsed angles of 78 to 92 are shown for clarity. All other elapsed angles produced curves with FPS values below those shown.

Constellation 24/3/1, which contains 8 satellites per orbital plane, was only checked at elapsed angles between 0 and 45 degrees since relative to the grid pattern this is the repetition angle. Note that this is confirmed by Figure 6 which shows that the curves for E.A. of 0 and 45 fall directly on top of each other. The fact that there are 8 satellites in each orbital plane at forty five degrees apart means that a satellite at the equator, for instance, will be replaced by one moving towards the same point in the equator after forty-five degrees.

The value of the largest angular distance to acquire three satellites (R_{max}) can be found in one of two ways. The first way is discussed in Section III and can be used when the number of satellites is relatively low (below 24 at least). It involves drawing the constellation being considered on a globe of some type and then finding the satellites surrounding the points of worst coverage. Using a closed form expression for R_{max}, a search is then made for the best inclination and so forth. The method is tedious and long. It can be computerized, however the runs tend to be very long and expensive due to the number of satellite combinations which could be involved in producing the worst case. This has been done by Walker[4]. The program, written in FORTRAN, is approximately 700 lines long.

The second method, employed in this work, is to take the Grader programs and for the prime candidates, say con-

stellations 24/24/14 and 24/3/1, to raise the visibility threshold angle (alpha). Runs are then made to determine what value of alpha gives no failures per sample (FPS = 0). This alpha value is a good approximation of R_{max}. Shown below are outputs of runs made for the constellation 24/3/1, Figure 8. The results can be found in Table 1 of Section I. In this case R_{max} is approximately 50 degrees.

```
          24/ 3/ 1
          ELAPSED ANGLE     =  20.00 (DEG)    <=======
          THREHSHOLD ANGLE  =  49.   (DEG)

INC   52.99 | FAIL/SAMPLE 0.0014
INC   53.49 | FAIL/SAMPLE 0.0011
INC   53.99 | FAIL/SAMPLE 0.0011
INC   54.49 | FAIL/SAMPLE 0.0007
INC   54.99 | FAIL/SAMPLE 0.0003

TOTAL NO. OF SAMPLES PER INCL. POINT = 2687
```

```
          24/ 3/ 1
          ELAPSED ANGLE     =  20.00 (DEG)    <=======
          THREHSHOLD ANGLE  =  50.   (DEG)

INC   52.99 | FAIL/SAMPLE 0.0000
INC   53.49 | FAIL/SAMPLE 0.0000
INC   53.99 | FAIL/SAMPLE 0.0000
INC   54.49 | FAIL/SAMPLE 0.0000
INC   54.99 | FAIL/SAMPLE 0.0000

TOTAL NO. OF SAMPLES PER INCL. POINT = 2687
```

Figure 8 - These outputs show how a threshold angle of 50° causes all failures in coverage to become zero at both 20 and 36 elapsed angles.

```
      24/ 3/ 1
      ELAPSED ANGLE     =  36.00 (DEG)     <=======
      THREHSHOLD ANGLE =  49.    (DEG)

INC   52.99 | FAIL/SAMPLE 0.0014
INC   53.49 | FAIL/SAMPLE 0.0011
INC   53.99 | FAIL/SAMPLE 0.0007
INC   54.49 | FAIL/SAMPLE 0.0003
INC   54.99 | FAIL/SAMPLE 0.0003

TOTAL NO. OF SAMPLES PER INCL. POINT = 2687
```

```
      24/ 3/ 1
      ELAPSED ANGLE     =  36.00 (DEG)     <=======
      THREHSHOLD ANGLE =  50.    (DEG)

INC   52.99 | FAIL/SAMPLE 0.0000
INC   53.49 | FAIL/SAMPLE 0.0000
INC   53.99 | FAIL/SAMPLE 0.0000
INC   54.49 | FAIL/SAMPLE 0.0000
INC   54.99 | FAIL/SAMPLE 0.0000

TOTAL NO. OF SAMPLES PER INCL. POINT = 2687
```

Figure 8 (continued) Note how there are no failures in coverage when the threshold angle is increased from 49 to 50 degrees.

Section III

Finding the Point of Worst Coverage

The method mentioned at the end of Section II and employed by Walker to find the point of worst coverage will be explained below. He gives a brief explanation of it in reference 2. The following comments might serve as a help to the reader since the approach will be covered in greater detail with some added modifications. This method was not used, however, in finding R_{max} as the author had initially intended. It was found to be somewhat cumbersome. In the end it does yield as precise a value as is desired. For the specific task at hand however, an approximate value is all that is required.

The point of worst coverage for a constellation is of interest because it is used as the measuring stick for performance. It represents the greatest angular distance (R_{max}) from the local vertical to a third subsatellite point (in the case of triple coverage) anywhere on the globe and at any elapsed angle of motion.

One might think that the point of worst coverage is the one with the greatest angular radius to three equidistant subsatellite points. Looking more closely, however, one finds that moving the user position closer to one of the

subsatellite points means that a circle of radius 'R' will no longer contain three subsatellite points. See Figure 9. Thus, the worst point is still to be found. We are assuming here a somewhat useful distribution where the satellites are not clustered.

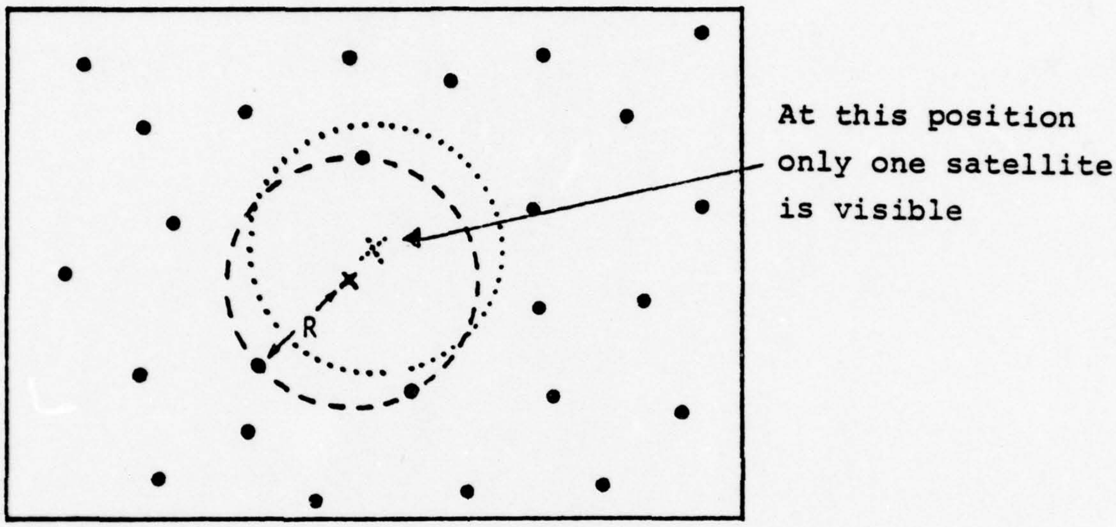

Fig. 9 The dotted circle shows worse coverage than the dashed circle.

For a true worst case position, then, one must usually find a point where three or more subsatellite points lie on the circumference of a circle with radius R_{max} and two subsatellite points lie within that radius. If there are less than three subsatellite points on the circumference, then there is a good chance that this is not the worst point. See Figure 10. Although in Figure 10 the user can see five satellites, if his angular visibility was only $R - \varepsilon$, he would only see two satellites, where ε is much smaller than R.

Thus R and not R - ε is required to have triple coverage. Note that for double coverage the user needs one subsatellite point within the circle and so on.

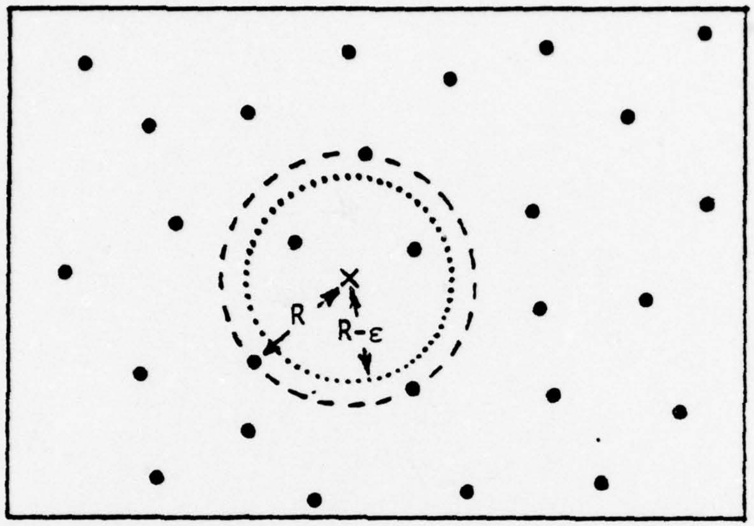

Fig. 10 Typical point of worst coverage at center of circle.

For a given constellation, then, one must first map the satellites (or subsatellite points) on a sphere. See Appendix B for a program which gives each individual satellite's latitude and longitude from the constellation parameters N/P/F, inclination, and elapsed angular motion. Then find the critical points which give maximum R values. In the case of 24/24/14, the critical points were though to occur at 0 and 15 degrees elapsed angle. Although this was later found to be incorrect, these elapsed angles will be used as example. At 0 degrees, the satellites producing maximum R are shown in Figure 11, and at 15 degrees, in Figure 12.

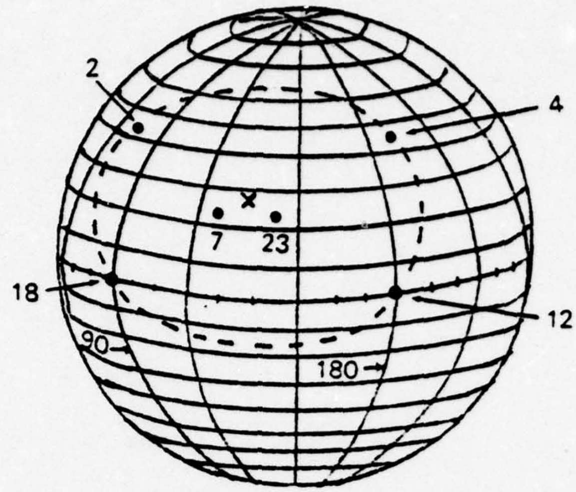

Figure 11 - Satellites making up what was thought to be the worst case of 24/24/14 at 0° E.A., 53° inclination.

Thus after the program 'Grader' has placed one in the vicinity of the optimal inclination, a graphical analysis to find the satellites making up the worst cases is performed. One can then generate a plot such as in Figure 13. The program used for this is explained in Appendix C. An even finer plot can be made to pinpoint the most favored inclination as shown in Figure 14 for this example, that inclination was 52.33°

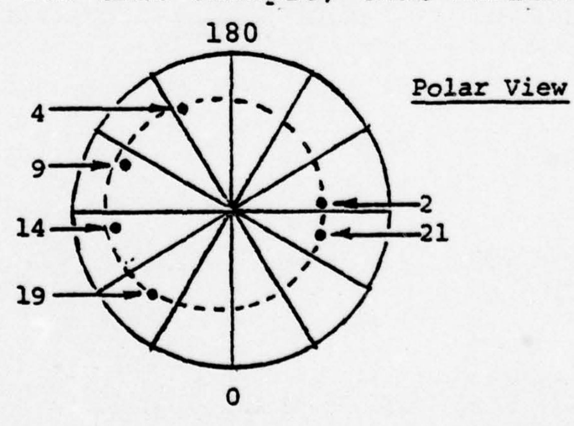

Figure 12 - Thought to be worst case satellites. Elapsed angle is 15 degrees.

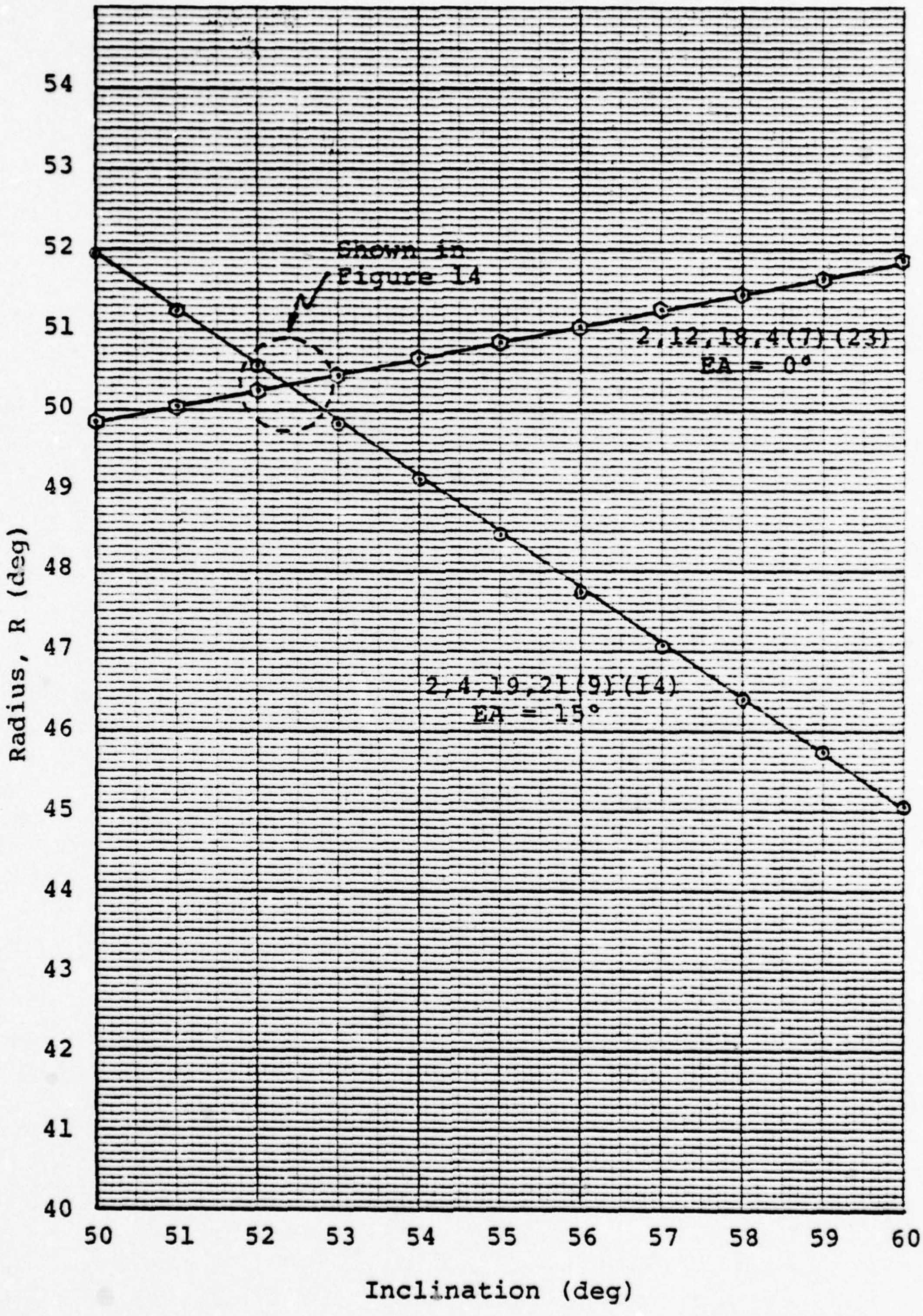

Figure 13 - A plot of R vs. inclination for what were thought to be the worst case elapsed angles of Constellation 24/24/14.

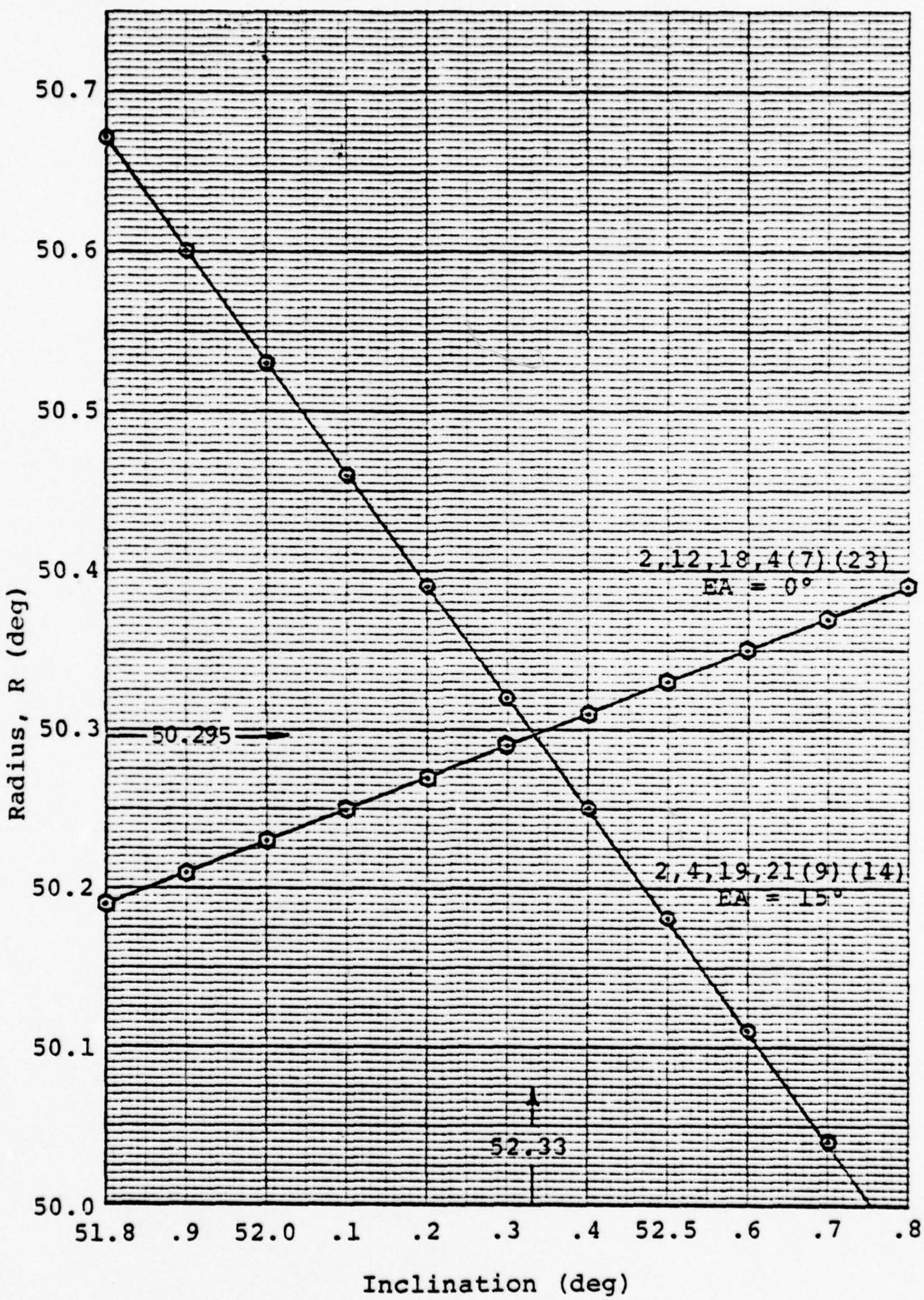

Figure 14 - Blow up of critical area for more accurate determination of optimal inclination for the example at hand.

The exception to the rule of finding the worst coverage is shown below in Figure 15. In this case there are no sub-satellite points within the circle with radius R_{max}. However there are enough satellites on the circumference that if the user moves in any direction he still has triple coverage.

Polar View

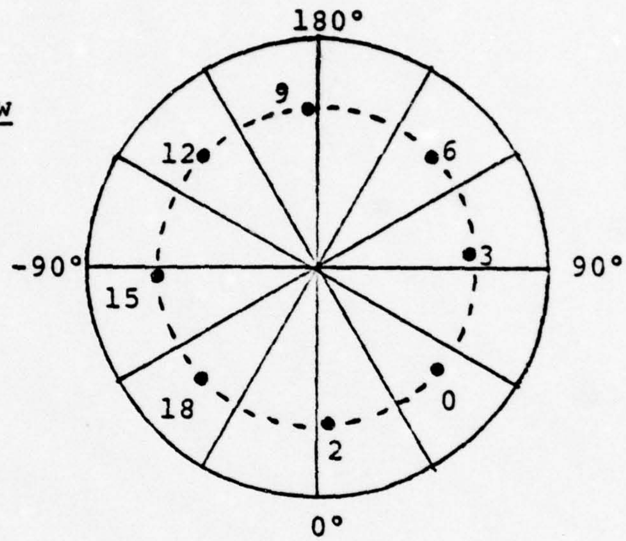

Figure 15 - Worst case for 24/24/8. An exception to the rule. E.A. is 60°.

In general, care must be taken not to miss areas of poor coverage when doing this kind of analysis by hand. The author had access to a slate-covered sphere with permanent latitude and longitude lines. This proved handy indeed. For measuring angular distances an ordinary compass was used with chalk on one side attached by means of rubber bands and a pencil strapped to the other end with the eraser acting as the pivot. Accuracies of $\pm 1°$ or better were found when the graphical value of R_{max} was compared to the computed value.

Section IV

The 'Grader' Program

The basic purpose of this program is to provide a quick tool for checking the performance of a constellation. It can help circumvent hours of tedious three dimensional plotting of twenty or so orbits for many possible phase angles and varying inclinations. The program has been called 'Grader' since it gives a figure of merit or grade to the candidate constellations.

It is a simple program which places a position vector in 2° steps in the latitudinal direction. It stops at each latitude and steps the position vector in the longitudinal direction. At each sample point it takes the dot product of this vector and subsatellite position vectors. The subsatellite vectors are arrived at from the corresponding longitude of ascending node (LAN) and nodal information of each satellite. A transformation is performed to get the information from this mode to rectangular coordinates for the purpose of taking the dot product. Dot products, instead of the corresponding angle, are then compared to a predetermined value. If the dot product is greater than this threshold value, the satellite is regarded as 'visible' to the vehicle (or user). A counter keeps track of the number of visible satellites. Whenever the total number of visible at a particular position is less than three, a triple coverage failure is noted.

At the end of all the sampling the total number of failures is divided by the total number of samples. Thus one arrives at the failure rate or failures per samples.

In order to save computer time, the analysis of coverage is done over a section of the globe and not over its entirety. Angles of 90° to 0° latitude and 0 to 180° in longitude were chosen. All the configurations encountered exhibited symmetry about the equator and also about a plane cutting perpendicular to the equator. This is the basis for the decision to look at only half of the northern hemisphere. In any case, half of the northern hemisphere is a large enough area for the accuracy desired in a preliminary study such as this.

Below is a sample output of the Grader for the 24/24/14 constellation at 80° elapsed angular motion. It is plotted in Figure 3.

```
          24/24/14
          ELAPSED ANGLE    =   80.00 (DEG)     <=======
          THRESHOLD ANGLE  =   45.00 (DEG)     (ALPHA)
     INC   50.99 | FAIL/SAMPLE 0.0468
     INC   51.99 | FAIL/SAMPLE 0.0446
     INC   52.99 | FAIL/SAMPLE 0.0457
     INC   53.99 | FAIL/SAMPLE 0.0468
     INC   54.99 | FAIL/SAMPLE 0.0506

     TOTAL NO. OF SAMPLES PER INCL. POINT = 2687
```

The Grader 1 program is almost identical to Grader only it handles cases where there are more than one satellite per orbit ($N > P$) such as 24/3/1. The two programs could have been made into one rather easily. Shown in Figure 17 is a sample output of the Grader 1 program.

```
24/ 3/ 1
ELAPSED ANGLE     =  40.00 (DEG)    <=======
THREHSHOLD ANGLE  =  45.   (DEG)

INC   50.99 | FAIL/SAMPLE 0.0521
INC   51.99 | FAIL/SAMPLE 0.0457
INC   52.99 | FAIL/SAMPLE 0.0337
INC   53.99 | FAIL/SAMPLE 0.0357
INC   54.99 | FAIL/SAMPLE 0.0360
INC   55.99 | FAIL/SAMPLE 0.0375
INC   56.99 | FAIL/SAMPLE 0.0446
INC   57.99 | FAIL/SAMPLE 0.0502

TOTAL NO. OF SAMPLES PER INCL. POINT = 2687
```

Figure 17 - Sample Output of the Grader-1 Program.

Another feature of the Grader program provided to save computer time decides which satellites can be taken out of the dot product part of the program. If one is analyzing half of the northern hemisphere, then there will be satellites which will never be visible to any of the position vectors. A quick survey at the beginning of the program is thus made to see if any satellites have too great a negative 'y' component or negative 'z' component thus making them ineffective to the area in question*. This is discussed in detail in Appendix A.

---

* Suggested by Robert White - C.S.Draper Laboratory

Appendix A

The 'Grader' Program in Detail

The 'Grader' program was written in the MAC language which lends itself well to vector operations. However, conversion to a more common language should be straightforward.

The input is shown in Figure A-1. It consists of:

a) 'longdelt' values (seldom change)

one card

b) inclination values
- initial
- increment
- final

c) N -- number of satellites

d) ALPHA -- threshold angle of visibility (deg)

one card

e) F -- phase angle number (0,1,2,3,..., N - 1)

f) EA -- elapsed angle (deg)

one card

g) F --

h) EA --

LONGDELT values are the incremental arcs in longitude through which the vehicle position vector is stepped. For example at 88° latitude, the increment in longitude for a net 2° motion is 1.0479 radians (or 60.04 degrees). At 86° latitude, .50588 radians (or 28.98 degrees) is the proper longitudinal step size and so on. A latitude of 90° does not have a corresponding step size since the calculations need only be done once. This is taken care of by the program. How the numbers were calculated will be explained later.

```
* COLLECT MLS1580.GRADER
* HEADING NETWORK   24/24/14   AND 24/24/5.
* GO
R *************************************************
R * LAST REVISION TO PROGRAM: MAY 13, 1977
R *************************************************
R          LONGDELT VALUES (45)
           1.04797767291
           5.05885909053E-01
           3.35553816957E-01
           2.51494055867E-01
           2.01367776409E-01 <—Note 1
           1.68093488520E-01
           1.44415920C71E-01
           1.26724659301E-01
           1.13019683190E-01
           1.02103263616E-01
           9.32142490412E-02
           8.58457373712E-02
           7.96471013844E-02
           7.43680877799E-02
           6.98253150938E-02
           6.58813537021E-02
           6.24311991331E-02
           5.93932596731E-02
           5.67031931536E-02
           5.43095946742E-02
           5.21709177248E-02
           5.02532359542E-02
           4.85285895684E-02
           4.69737457550E-C2
           4.55692571675E-02
           4.42987382150E-02
           4.31483027015E-02
           4.21061225043E-02
           4.11620781126E-02
           4.03074796417E-02
           3.95348424624E-C2
           3.88377055657E-02
           3.82104836699E-02
           3.76483462053E-C2
           3.71471178957E-02
           3.67031968401E-02
           3.63134869029E-02
           3.59753419082E-02
           3.56865196697E-02
           3.54451443050E-02
           3.52496756117E-02
           3.50988845493E-02
           3.49918340816E-C2
           3.49278648186E-02
           3.49065850398E-02
R         INCIN,INCDELT, INCFIN, N, ALPHA
          51,      1,      55,   24, 45
R         F,EA
           14,78
           14,80
           14,82
           14,84
           14,86
           14,88
           14,90
           14,92
            0,0
           40,     5,      70,   24, 45
            5,0
```

Figure A-1 - Listing of input for the Grader program.

INCIN, INCDELT, INCFIN represent the initial inclination, inclination increment, and final inclination respectively. Suggested values are: 40, 5, 70 (coarse plotting)

and 50, 1, 60 (fine plotting).

F, EA, representing F value or phase angle value and the elapsed angle, are in the inner most loop with a read statement at the beginning of the loop. One can have any combination whatsoever. The elapsed angle (EA) is the angular motion of all the satellites in their respective orbital planes.

<u>Alpha</u>

Alpha is the threshold angle. This earth central angle between user and satellite defines visibility for the program. If a vehicle position vector is found at any angle greater than this amount it is rendered nonvisible. For 24/24/14, 45° was found to be a good number.

It is important to choose an alpha value that is less than the expected performance of the constellation so that the percentage of failed samples will not be zero. It is equally important to use this threshold value for all the runs for a given N value so that a meaningful comparison can be made.

<u>More on Longdelt</u>

The value of this 2° net longitudinal movement, longdelt, was put in to compensate for the fact that a degree of longitude at the equator and a degree of longitude near the north

pole represent radically different angular values on the surface of the globe. If one were to disregard this, one would find that a disproportionate number of samples would be taken near the north polar area.

Let us represent the vehicle position vector in rectangular coordinates. To do this a transformation vector is needed to transform from ℓ, L to x, y, z. This then is

$$\hat{p}_0 = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} = \begin{bmatrix} \cos \ell_0 \cdot \cos L_0 \\ \cos \ell_0 \cdot \sin L_0 \\ \sin \ell_0 \end{bmatrix}$$

where $L_0 = 0$.

Then another position vector, two degrees away from $\hat{p}_0$ can be defined as

$$\hat{p}_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} \cos \ell_1 \cos L_1 \\ \cos \ell_1 \sin L_1 \\ \sin \ell_1 \end{bmatrix}$$

where $\begin{cases} \ell_1 = \ell_0 \\ L_1 = ? \end{cases}$

Since they are both unit vectors

$$\cos \theta = \hat{p}_0 \cdot \hat{p}_1$$

$$\text{or } \cos 2° = \cos \ell_0 \cos L_0 \cos \ell_1 \cos L_1$$

$$+ \cos \ell_0 \sin L_0 \cos \ell_1 \sin L_1$$

$$+ \sin \ell_0 \sin \ell_1 .$$

Substituting ℓ_0 for ℓ_1 and 0 for L_0 yields:

$$\cos 2° = \cos \ell_0 \cos \ell_0 \cos L_1 + \sin \ell_0 \sin \ell_0$$

$$\text{so that } \cos L_1 = \frac{\cos 2° - \sin^2 \ell_0}{\cos^2 \ell_0}$$

$$L_1 = \cos^{-1}\left[\frac{\cos 2° - \sin^2 \ell_0}{\cos^2 \ell_0}\right]$$

For 80° latitude (ℓ_0),

$$L_1 = \cos^{-1}\left[\frac{\cos 2° - \sin^2 80°}{\cos^2 80°}\right]$$

$= 11.54°$ (.2014 radians, Note 1 in Figure A-1).

Thus at 80° latitude, the vehicle position vector is placed at 11.54 degree increments starting with 0°, 11.54°, 23.08,...

At each latitude then, a corresponding incremental step in longitude is made. This gives a close enough approximation to the desired homogeneous sampling pattern.

OUTPUT

The output is straightforward, a sample of which is shown again below for convenience. The N/P/F along with elapsed angle describe the constellation uniquely once an inclination is chosen. The 'threshold angle' is simply alpha as defined above. The 'total number of samples per inclination point' is the number used to calculate the 'FAIL/SAMPLE' number. It is not computed in each run (since it remains the same for this grid) but was found once by means of a counter in the program. The total number of failures is divided by the total number of samples to get F/S.

```
NETWORK  24/24/14

R ***************************************************************
R * LAST REVISION TO PROGRAM: MAY 12, 1977
R ***************************************************************
R          LONGDELT VALUES (43)
R         INCLIN, INCDELT, INCFIN, N, ALPHA
R          51.      1.        55.  24. 45
R          F.EA
              24/24/14
              ELAPSED ANGLE =  73.[illegible] (DEG)    <========
              THRESHOLD ANGLE =  45.00 (DEG)   (ALPHA)
         INC  50.99 | FAIL/SAMPLE 0.0442
         INC  51.99 | FAIL/SAMPLE 0.0427
         INC  52.99 | FAIL/SAMPLE 0.0454
         INC  53.99 | FAIL/SAMPLE 0.0451
         INC  54.99 | FAIL/SAMPLE 0.0529

         TOTAL NO. OF SAMPLES PER INCL. POINT = 2567
```

Disregarding Satellites Unreachable to the Sampling Area

The procedure of disregarding satellites which cannot be 'seen' by the sampling position vectors is simple and, as mentioned in the main test, can save computer time. What is done here is to look at the components of the satellite position vector and see if either the y component or the z component is sufficiently negative to render it non-visible. Assuming that the sampling area is the one used in this study, then

a) if $RR_j < -\sin(\alpha)$,
b) or if $RR_k < -\sin(\alpha)$

the satellite is not visible to the sampling area.

where RR_j = satellite y component

RR_k = satellite z component

α = visibility threshold angle setting.

See notes 1 and 2 of the program, Figure A-2.

Transformation of the Satellite Position Vector from Keplerian Elements to Rectangular Coordinates

In order to see whether a satellite is within a specific angle to a sample point, the Keplerian elements are transformed to rectangular coordinates on a unit sphere. The sample point, which is in latitude, longitude coordinates is then also transformed to rectangular coordinates. The scalar product of the two can then be taken and the angle checked.

```
*000010 PRODMAC    MLS1580.GRADER
M000020            INDEX A,I,J,K,M
M000025            DIMENSION (CO,25),(SO,25)
M000030            DIMENSION (COCU,25),(SOCU,25),(SOSU,25),(COSU,25),(SU,25)
M000040            DIMENSION (RR,25X3),(LONGDELTR,46),(CL,46),(SL,46)
R000050 *******************************************************************
R000060 *  GRADER.MAC                                                     *
R000070 *       - ANALYZES THE COVERGAE PRODUCED BY N SATELLITES OVER A SPECIF-*
R000080 *         IED PORTION OF THE GLOBE:                               *
R000090 *                2 DEGREE  LATTITUDE INCREMENT,                   *
R000100 *            AND 2 DEGREES INC IN LONGITUDINAL DIRECTION (NOT IN LONG) *
R000110 *       - THE SATELLITE NETWORK IS POSTULATED BY SUAREZ.          *
R000120 * LONGMIN = MINIMUM LONGITUDE DESIRED. 0 OR 90 SUGGESTED.         *
R000130 * LONGMAX = MAXIMUM LONGITUDE DESIRED. 90 OR 180 SUGGESTED.       *
R000140 *                                                                 *
R000150 *      - THIS PROGRAM ASSUMES CIRCULAR UNPERTURBED ORBITS.        *
R000160 * ALPHA = MAXIMUM EARTH CENTRAL ANGLE BETWEEN VEHICLE AND THE SATEL- *
R000170 *         LITE WHICH IT IS TRYING TO SEE.  (DEG.)                 *
R000180 *       = IT SHOULD BE SET TO A VALUE LESS THAN THAT EXPECTED FOR *
R000190 *         SUCCESSFUL TRIPLE COVERAGE.                             *
R000200 * OMEGA = LONGITUDE OF ASCENDING NODE     (DEG).                  *
R000210 * U     = NODAL ANGLE OF A SATELLITE IN ITS ORBIT (DEG).          *
R000220 * RR    = PSEUDO COMPONENTS OF SATELLITE POSITION VECTOR IN X,Y,Z *
R000230 *         INERTIAL FRAME                                          *
R000240 * P     = THE POSTION VECTOR OF THE VEHICLE (P0,P1,P2).           *
R000250 * INC   = INCLINATION OF ORBIT (DEG).                             *
R000260 * THETA = THE ANGLE BETWEEN SATELLITE AND VEHICLE.                *
R000270 * INCIN = INTIAL INCLINATION (DEG).                               *
R000280 * INCDELT = INCLINATION INCREMENT (DEG).                          *
R000290 * INCFIN= FINAL INCLINATION FOR THE LOOP.(DEG).                   *
R000300 * 2687  = NO. OF POSITIONS THE VEHICLE IS PLACED IN FOR SAMPLING. *
R000310 * N     = NO. OF SATELLITES.                                      *
R000320 *******************************************************************
M000321 BEGIN      CL   =0,    SL   =1
S000322              45          45
M000323            DO TO END11 FOR M = 44(-1)0
M000324            LATR = M 0.034907
M000325            CL   = COS(LATR), SL   = SIN(LATR)      } Note 6
S000326              M                M
M000327 END11      READ LONGDELTR
S000328                           M
```

Figure A-2 - Listing of the Grader program for cases where N=P. "R" on the far left represents a comment.

```
M000330 READ1    READ INCIN,INCDELT,INCFIN,N,ALPHA
M000340          INCINR   = INCIN   DEGTORAD
M000350          INCDELTR = INCDELT DEGTORAD
M000360          INCFINR  = INCFIN  DEGTORAD - 0.001
M000370          ALPHAR   = ALPHA   DEGTORAD
M000380          CT = COS(2 DEGTORAD)
M000390          CA = COS(ALPHAR)                     } Note 2
M000400          SA = SIN(ALPHAR)
M000410          LANDELT = 2 PI/N
M000420          DO TO END02 FOR I=0(1)(N-1)
M000430          OMEGAR = I LANDELT
M000440 END02    CO  = COS (OMEGAR), SO  = SIN(OMEGAR)
S000450            I                  I
M000500 READ2    READ F,EA
M000505          IF F = 0, GO TO READ1
M000510          EAR      = EA      DEGTORAD
M000520          NODEDELT = F LANDELT
M000530          DO TO ENDOO FOR I = 0(1)(N-1)
M000540          UR = EAR + I NODEDELT


M000550          SU = SIN(UR)
S000560            I
M000570          CU = COS(UR)                                                   Note 3
M000580 ENDOO    COCU =CO CU, SOCU =SO CU, SOSU =SO SU ,COSU =CO SU
S000590             I  I          I  I          I  I  I        I  I  I
M000680          PRINT FORMAT 15,N,N,F,EA,ALPHA
M000690          FORMAT 15
E000691              $$/$$/$$
M000700              ELAPSED ANGLE   = $$$.$$ (DEG)   <=======
S000705              THRESHOLD ANGLE = $$$.$$ (DEG)   (ALPHA)
R000710 ----------------------------------------------------------------------
M000720          DO TO ENDO FOR INCR = INCINR(INCDELTR)INCFINR
M000730          M=0, NP=N, FAILD = 0
M000740          CI = COS(INCR)                                  | Note 4
M000750          SI = SIN(INCR)
```

Figure A-2 (continued) The Grader program.

```
R000760                  --------------- SATELLITE POSITION LOOP ------------------
M000770            DO TO END1 FOR A = 0(1)(N-1)
M000780            I = 3 M
M000790            RR   = COCU - SCSU CI                                          Note 5
S000800              I         A       A
M000810            J = I + 1
M000820            RR   = SOCU  + COSU  CI
S000830              J         A        A
M000840            K = I + 2
M000850            RR   = SU  SI
S000860              K       A
M000870            IF RR < -SA, NP=NP-1,GO TO END1                                Note 1
S000880                 J
M000890            IF RR < -SA, NP=NP-1,GO TO END1
S000900                 K
M000910            M = M+1
M000920 END1       NIL = 0
R000930                  ----------- END SAT. POSITION LOOP ------------
M000940            NPP = 3 (NP-1)
R000950                  --------------- LATITUDE LOOP ---------------------
M000960            DO TO END43 FOR M = 45(-1)0
M000970            P   = SL                                                       Note 7
S000980             2     M
M000990            DO TO END43 FOR LONGR = 0(LONGDELTR )3.1416
S001000                                                M
M001010            P   = CL  COS(LONGR)                                           Note 8
S001020             0     M
M001030            P   = CL  SIN(LONGR)
S001040             1     M
M001050            ZUM = 0
M001060            DO TO END5 FOR I = 0(3)(NPP)
E001070                  -    -
M001080            DOT = RR  . P
S001090                   I
M001100            IF DOT  > CA, ZUM = ZUM + 1
M001105            IF ZUM = 3, GO TO END43
M001110 END5       NIL = 0
M001120            IF ZUM < 3, FAILD=FAILD + 1
M001130 END43      NIL = 0
R001140            ----------------- END LONGITUDE AND LATTITUDE LOOPS-----
```

Figure A-2 (continued) The Grader program.

```
M001150              FPS = FAILD/2687
M001160              INC = INCR RADTODEG
MC01170              PRINT FORMAT 62, INC, FPS
M001180              FORMAT 62
M001190              INC $$$.$$ | FAIL/SAMPLE $.$$$$
M001200 ENDO         NIL = 0
M001210              PRINT FORMAT 80
M001220              FORMAT 80
E001230
M001240              TOTAL NO. OF SAMPLES PER INCL. POINT = 2687
M001250              PRINT SKIP
M001260              GO TO READ2
M001270              START AT BEGIN
```

Figure A-2 (continued) The Grader program.

The transformation of the satellite elements then can be done by using a simplified version of the R matrix shown below*

$$\begin{bmatrix} q_x \\ q_y \\ q_z \end{bmatrix} = \underline{R} \begin{bmatrix} q_\xi \\ q_\eta \\ q_\zeta \end{bmatrix}$$

The figure shown shows the vector components. The R matrix is:

$$\begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \\ u_1 & u_2 & u_3 \end{pmatrix}$$

$$\ell_1 = \cos \Omega \cos \omega - \sin \Omega \sin \omega \cos i$$

$$\ell_2 = -\cos \Omega \sin \omega - \sin \Omega \cos \omega \cos i$$

$$\ell_3 = \sin \Omega \sin i$$

$$m_1 = \sin \Omega \cos \omega + \cos \Omega \sin \omega \cos i$$

$$m_2 = -\sin \Omega \sin \omega + \cos \Omega \cos \omega \cos i$$

$$m_3 = -\cos \Omega \sin i$$

$$n_1 = \sin \omega \sin i$$

$$n_2 = \cos \omega \sin i$$

$$n_3 = \cos i$$

---

* Battin, Astronautical Guidance, p. 18

However, since one is considering simply a position on the unit sphere, the vector components q_η and q_ζ can be set to zero and q_ξ set of 1. Thus

$$\begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix} = \begin{bmatrix} \ell_1 \\ m_1 \\ n_1 \end{bmatrix}$$

Notes 3, 4 and 5 on the program source show how RR_i, RR_j, and RR_k are derived corresponding to ℓ_1, m_1, and n_1 respectively.

<u>Transformation of the User Position Vector to Rectangular Coordinates</u>

Since latitude and longitude angles lend themselves readily to a grid sampling pattern on a unit sphere, they are employed here. After the corresponding increment is made, they are transformed to rectangular coordinates.

The transformation is then,

$$\begin{bmatrix} P_0 \\ P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos(\text{lat}) \cos(\text{long}) \\ \cos(\text{lat}) \sin(\text{long}) \\ \sin(\text{lat}) \end{bmatrix}$$

This corresponds to notes 6, 7, and 8 of the program listing.

Other features which can save computer time have been incorporated and will be discussed in Appendix A. By incorporating all these, the author was able to cut the cost per run by more than 60%.

Figure A-3 shows a listing of the Grader-1 program. Only the part which is different is shown for brevity. The rest is exactly the same for both programs.

```
*000010 PRODMAC   MLS1580.GRADER1
M000020           INDEX A,I,J,K,M,O
M000030           DIMENSION (COCU,24),(SOCU,24),(SOSU,24),(COSU,24),(SU,24)
M000040           DIMENSION (RR,24X3),(LONGDELTR,46),(CL,46),(SL,46)
R000050 ******************************************************************
R000060 *  GRADER1.MAC                                                   *
R000070 *       - ANALYZES THE COVERGAE PRODUCED BY N SATELLITES OVER A SPECIF-*
R000080 *         IED FORTION OF THE GLOBE:                                *
R000090 *               2 DEGREE  LATTITUDE INCREMENT,                     *
R000100 *            AND 2 DEGREES INC IN LONGITUDINAL DIRECTION (NOT IN LONG) *
R000110 *                                                                  *
R000120 *                                                                  *
R000130 *       - THIS PROGRAM ASSUMES CIRCULAR UNPERTURBED ORBITS.        *
R000140 * ALPHA = MAXIMUM EARTH CENTRAL ANGLE BETWEEN VEHICLE AND THE SATEL- *
R000150 *         LITE WHICH IT IS TRYING TO SEE.  (DEG.)                  *
R000160 *       = IT SHOULD BE SET TO A VALUE LESS THAN THAT EXPECTED FOR  *
R000170 *         SUCCESSFUL TRIPLE COVERAGE.                              *
R000180 * OMEGA = LONGITUDE OF ASCENDING NODE     (DEG).                   *
R000190 * U     = NODAL ANGLE OF A SATELLITE IN ITS ORBIT (DEG).           *
R000200 * RR    = PSEUDO COMPONENTS OF SATELLITE POSITION VECTOR IN X,Y,Z  *
R000210 *         INERTIAL FRAME                                           *
R000220 * P     = THE POSTION VECTOR OF THE VEHICLE (P0,P1,P2).            *
R000230 * INC   = INCLINATION OF ORBIT (DEG).                              *
R000240 * THETA = THE ANGLE BETWEEN SATELLITE AND VEHICLE.                 *
R000250 * INCIN = INTIAL INCLINATION (DEG).                                *
R000260 * INCDELT = INCLINATION INCREMENT (DEG).                           *
R000270 * INCFIN= FINAL INCLINATION FOR THE LOOP.(DEG).                    *
R000280 * 2687  = NO. OF POSITIONS THE VEHICLE IS PLACED IN FOR SAMPLING.  *
R000290 * N     = NO. OF SATELLITES.                                       *
R000300 ******************************************************************
M000310 BEGIN     CL  =0,    SL  =1
S000320             45         45
M000330           DO TO END11 FOR M = 44(-1)0
M000340           LATR = M 0.034907
M000350           CL  = COS(LATR), SL  = SIN(LATR)
S000360             M               M
M000370 END11     READ LONGDELTR
S000380                          M
```

Figure A-3 - Listing of the Grader-1 program for constellations where N>P. Only the first part is listed. The remainder is the same as is found in Grader.

```
M000390 READ1     READ INCIN,INCDELT,INCFIN,N,ALPHA
M000400           INCINR   = INCIN   DEGTORAD
M000410           INCDELTR = INCDELT DEGTORAD
M000420           INCFINR  = INCFIN  DEGTORAD - 0.001
M000430           ALPHAR   = ALPHA   DEGTORAD
M000440           CT = COS(2 DEGTORAD)
M000450           CA = COS(ALPHAR)
M000460           SA = SIN(ALPHAR)
M000470 READ2     READ O,F,EA
M000480           IF O<1, GO TO READ1
M000490           RUN = RUN + 1
M000500           EAR    = EA DEGTORAD
M000510           PHASE  = F 2 PI/N
M000520           NPO    = N/O
M000530           LANDELT  = 2 PI/O
M000540           NODEDELT = 2 PI/NPO
R000550           ------------------------------
M000560           I=0
M000570           IF RUN = 1, PRINT FORMAT 12
M000580              FORMAT 12
M000581             L.A.N.        NODE  (RADIANS)  FOR RUN ONE ONLY.
M000590           DO TO ENDOO FOR ORBIT = 0(1)(O-1)
M000600           OMEGAR = LANDELT ORBIT
M000610           CO = COS (OMEGAR), SO = SIN (OMEGAR)
M000620           DO TO ENDOO FOR J = 0(1)(NPO-1)
M000630           UR = EAR + ORBIT PHASE + J NODEDELT
M000640           IF RUN=1, PRINT FORMAT 14, OMEGAR, UR
M000650             FORMAT 14
M000660             $$.$$$  |   $$.$$$$
M000670           SU = SIN(UR), CU = COS(UR)
S000680             I
M000690           COCU =CO CU, SOCU =SO CU, SOSU =SO SU ,COSU =CO SU
S000700               I            I             I      I      I      I
M000710 ENDOO     I = I + 1
M000720           PRINT SKIP
M000730           PRINT FORMAT 15,N,O,F,EA,ALPHA,SP3
M000740           FORMAT 15
E000750               $$/$$/$$
M000760               ELAPSED ANGLE     = $$$.$$ (DEG)    <=======
S000770               THRESHOLD ANGLE =  $$.    (DEG)
R000780 -----------------------------------------------------------------------
```

Figure A-3 (continued) The Grader-1 program.

Appendix B

Network Orbit Plotting Program and Derivation

After one has an idea of a suitable inclination for a given configuration of N satellites, the question of plotting the network and finding the point of worst coverage arises. After looking at different ways of representing the different ways of representing the different orbits and the satellites within those orbits, this method was found to be the best to obtain the quickest spherical plot. From the plot one can then find the trouble areas and the resulting maximum radius (R_{max}) for triple coverage.

The program deals with both types of constellations (studied in this paper), the type where N = P and the type where N > P, where P is the number of orbits. The N > P case will be used as an example here. Basically there are four steps to the program:

1) Input describing the constellation and run,
2) the computation of LAN and node for each satellite,
3) the transformation to rectangular coordinates,
4) and the transformation to latitude and longitude.

Figure B-1 shows the source programs. It was developed on a Hewlett-Packard 9820 desk calculator. Figure B-2 shows the output of a run used in the analysis of the 24/3/1 con-

```
0: FXD 2; ENT "MONTH.DATE",R81
1: PRT "       MONTH.DATE",R81; PRT "     1977"
2: FXD 0; ENT "SATS",A
3: ENT "PLANES",R3
4: ENT "PHASE NO.(F)",R6; R6*360/a- R0
5: PRT "..... N/P/F....",A,R3,R6
6: FXD 1; ENT "INCLINATION",B; PRT "INCL.",B
7: ENT "NO.OF PTS",R20; PRT "NO.OF PTS",R20
8: ENT "PLOT ANGLE INCR.",R21; PRT "PLOT ANGLE INCR.",R21
9: 360/A→C;IF A>R3;360/R3→C;a/R3→R7
10: COS B→R1;SIN B→R2;0→R4→R10→R110
11: "10"; SPC 1; PRT "SAT.NO.---------",R10;0→R100
12: IF A>R3;GTO "13"
13: R10*C→R11; R10*R0→R12;GTO "15"
14: "13"; R4*C→R11; R4+R0*360/R7→R12
15: "15"; R12-360→R13; IF 0>R13;GTO "17"
16: R13→R12; GTO "15:
17: "17"; PRT "LAN,NODE,R11,R12
18: Prt "LONG,LAT"
19: "20";COS R11*COS R12-SIN R11*SIN R12*R1→R31
20: IF R31=0; .00001→R31
21: SIN R11*CIS R12+COS R11*SIN R12*R1→R32
22: SIN R12*R2→R33
23: ATN (R32/R31)→R41
24: IF 0>R31; R41+180→R41
25: IF 0>R41; 360+R41→R41
26: PRT R41; PRT ASN R33
27: R12+R21→R12; R100+1→R100
28: IF R20>R100; GTO "20"
29: R110+1→R110; IF R110>R7-1; 0→R110; R4+1→R4
30: R10+1→R10;IF≤R10 A-1; GTO "10"
31: SPC 7; END
```

Figure B-1 - Program giving latitude and longitude in time of multisatellite constellations.

```
    MONTH.DATE
          5.02
          1977
..... N/P/F .....
            24
             3
             1
INCL..
          53.0
NO.OF PTS
           5.0
PLOT ANGLE INCR.
          10.0

SAT.NO.--------
           0.0
LAN,NODE
           0.0
           0.0
LONG,LAT
           0.0
           0.0
           6.1
           8.0
Note B-1  12.4
          15.9
          19.2
          23.5
          26.8
          30.9

SAT.NO.--------
           1.0
LAN,NODE
           0.0
          45.0
LONG,LAT
          31.0
          34.4
          40.7
          40.9
          52.2
          46.4
          66.0
          50.5
          81.7
          52.7
```

```
SAT.NO.--------
           2.0
LAN,NODE
           0.0
          90.0
LONG,LAT
          90.0
          53.0
         106.3
          51.9
         121.2
          48.6
         133.8
          43.8
         144.4
          37.7

SAT.NO.--------
           3.0
LAN,NODE
           0.0
         135.0
LONG,LAT
         149.0
          34.4
         157.1
          27.3
         164.3
          19.7
         170.8
          11.9
         177.0
           4.0

SAT.NO.--------
           4.0
LAN,NODE
           0.0
         180.0
LONG,LAT
         180.0
           0.0
         186.1
          -8.0
         192.4
         -15.9
         199.2
         -23.5
         206.8
         -30.9
```

```
SAT.NO.--------
           5.0
LAN,NODE
           0.0
         225.0
LONG,LAT
         211.0
         -34.4
         220.7
         -40.9
         232.2
         -46.4
         246.0
         -50.5
         261.7
         -52.7

SAT.NO.--------
           6.0
LAN,NODE
           0.0
         270.0
LONG,LAT
         270.0
         -53.0
         286.3
         -51.9
         301.2
         -48.6
         313.8
         -43.8
         324.4
         -37.7

SAT.NO.--------
           7.0
LAN,NODE
           0.0
         315.0
LONG,LAT
         329.0
         -34.4
         337.1
         -27.3
         344.3
         -19.7
         350.8
         -11.9
         357.0
          -4.0
```

Figure B-2 Part of the output for plot of 24/3/1 on a globe.

stellation. Let us first look at the output closely to understand the purpose of the program. Then the algorithm which gives that output will be discussed.

The Output

Month-Date - 5.02 - 1977: the program's first question to the user is the date. The number 5.02 was typed in referring to May 2.

N/P/F - 24,3,1: This refers to the constellation 24/3/1.

INCL. - 53.0: The inclination is requested by the program and then printed.

NO. OF PTS. - 5.0: The number of points to be plotted for each satellite can be expressed also as the number of nodal positions a satellite will inhabit in its orbit.

PLOT ANGLE INCR. - 10.0: The plot angle increment refers to the increment in nodal angle for these positions. In this case, therefore, we are looking at node angles of 0,10,20,30, and 40°.

SAT. NO. - 0.0: This refers to the satellite number. Zero is used because it makes the programming simpler.

LAN, NODE - 0.0 - 0.0: These two numbers represent the longitude of ascending node and the node angle of the n-th satellite.

LONG, LAT - 0.0, 0.0; 6.1, 8.0;...: This is the desired output and it is the longitude and latitude of the satellite at the node angles requested. See note B-1.

One is then ready to plot.

The following is a description of the variables found in the program.

HP Notation	Regular Notation	Description
A	N	the number of satellites
B	i	inclination (this assumed the same for all the planes)
C	$\Delta\Omega$	increment in longitude g ascending node from plane to plane
RO	$\Delta\omega$	increment in node angle ($\Delta\omega = F \times 360/N$)
R1	cos i	
R2	sin i	
R3	P	number of planes in which the satellites are distributed
R4		the number of individual plane (0,1,..., P - 1)
R6	F	the phase angle number
R7	NPO	number of satellites per orbit NPO = N/0
R8	ϕ	phase angle between a given satellite and the satellite in the next adjacent longitude of ascending node, $\phi = F \times 360/0$
R10	i	satellite number
R11	Ω_i	longitude of ascending node of ith satellite
R12	ω_i	node of the ith satellite

HP Notation	Regular Notation	Description
R13		dummy variable for R12
R20		the number of points to be plotted per satellite
R21		increment through which all the satellites are to be moved in their respective orbits
R31	x	x component of the ith satellite
R32	y	y component of the ith satellite
R33	z	z component of the ith satellite
R41	l	longitude of the ith satellite
R81		month.date in one variable
R110	j	jth satellite of a given orbital plane when $N > P$ (0,1,2,.., NPO - 1)

Appendix C

Program for Plotting R vs. Inclination For a Given Group of Satellites

Once the satellites surrounding the worst coverage points have been identified as in Section III, the program for plotting R versus inclination is very helpful. This was shown in Section III with the example of the constellation 24/24/14 where it was used to produce Figures 13 and 14. Although it was not used in obtaining the final results given in Table 1 (p. 9), for better accuracy something similar might be employed.

Listed below are the steps in the program originally written for a large desk calculator (Hewlett-Packard 9820).

```
[illegible]
[illegible]
PRT "[illegible]
"⊢
1:
FXD 0;ENT "SAT N
OS?",R8,R9,R10;
PRT "     SAT NOS
....",R8,R9,R10⊢
2:
FXD 2;ENT "MONTH
.DATE?",R6;PRT "
      DATE",R6;
PRT "
1977"⊢
3:
ENT "LAN,NODE? S
AT A",R21,R41;
PRT "LAN,NODE-SA
T A",R21,R41⊢
4:
ENT "LAN,NODE-SA
T B",R22,R42;
PRT "SAT B",R22,
R42⊢
5:
ENT "LAN,NODE SA
T C",R23,R43;
PRT "SAT C",R23,
R43⊢
6:
ENT "INITIAL INC
?",B;ENT "INC.DE
LT?",C;ENT "FINA
L INC?",R7⊢
7:
PRT "IN.,DELT,&F
INAL INC";PRT "I
NCLINATION",B,C,
R7⊢
```

```
8:
ENT "ELAPSED NOD
E?",R13;PRT "ELA
PSED NODE",R13⊢
9:
0→A;SPC 1;PRT "L
ONGS,LATS";B→R1⊢
10:
"20";R(21+A)→R11
;R(41+A)+R13→R12
;COS R11*COS R12
-SIN R11SIN R12*
COS R1→R(31+A)⊢
11:
IF R(31+A)=0;1E-
9→R(31+A)⊢
12:
SIN R11*COS R12+
COS R11*SIN R12*
COS R1→R(71+A)⊢
13:
SIN R12*SIN R1→R
(91+A)⊢
14:
ATN (R(71+A)/R(3
1+A))→R(51+A)⊢
15:
IF 0>R(31+A);R(5
1+A)+180→R(51+A)
⊢
16:
IF 0>R(51+A);360
+R(51+A)→R(51+A)
⊢
17:
PRT R(51+A);ASN
R(91+A)→R(61+A);
PRT R(61+A)⊢
18:
A+1→A;IF A≤2;
GTO "20"⊢
19:
"-----------"⊢
20:
ACS (R32R33+R72R
73+R92R93)→R2⊢
21:
ACS (R31R33+R71R
73+R91R93)→R14⊢
22:
ACS (R31R32+R71R
72+R91R92)→R15⊢
23:
ACS ((COS R2-
COS R14*COS R15)
/(SIN R14*SIN R1
5))→R3⊢
24:
ACS ((COS R14-
COS R2*COS R15)/
(SIN R2*SIN R15)
)→R4⊢
25:
ACS ((COS R15-
COS R2*COS R14)/
(SIN R2*SIN R14)
)→R5⊢
26:
ATN (TAN (R2/2)*
1/COS (.5(R4+R5-
R3)))→R105⊢
27:
PRT "FOR INC.=..
....",R1;PRT "RA
DIUS = ......",R
105;SPC 2⊢
28:
R1+C→R1;IF R1≤R7
;0→A;GTO "20"⊢
29:
SPC 6⊢
```

Below is an explanation of the variables found in the program. The units are in degrees unless otherwise specified.

Program Variable	Corresponding Variable	Comments
A		counter (no units)
B	i_0	initial inclination
C	Δi	increment in inclination
R1	i	inclination
R2	a	side a of the spherical triangle
R3	A	angle A opposite to side a on spherical triangle
R4	B	angle B opposite to side b on spherical triangle
R5	C	angle C opposite to side c on spherical triangle
R6		month.date of the run
R7	i_f	final inclination
R11	Ω_i	longitude of ascending node for the i-th satellite
R12	ω_i	node angle of the i-th satellite
R13	EA	elapsed node under investigation
R14	b	side b of the spherical triangle
R15	c	side c of the spherical triangle
R21	Ω_A	longitude of ascending node of satellite A
R22	Ω_B	longitude of ascending node of satellite B
R23	Ω_C	longitude of ascending node of satellite C
R31	x_A	component in the x direction of satellite A (length units)

Program Variable	Corresponding Variable	Comments
R32	x_B	component in the x direction of satellite B (length units)
R33	x_C	component in the x direction of of satellite C (length units)
R41	ω_A	node angle of satellite A at 0 E.A.
R42	ω_B	node angle of satellite B at 0 E.A.
R43	ω_C	node angle of satellite C at 0 E.A.
R51	ℓ_A	longitude of satellite A
R52	ℓ_B	longitude of satellite B
R53	ℓ_C	longitude of satellite C
R61	L_A	latitude of satellite A
R62	L_B	latitude of satellite B
R63	L_C	latitude of satellite C
R71	y_A	component in the y direction of satellite A (length units)
R72	y_B	component in the y direction of satellite B (length units)
R73	y_C	component in the y direction of satellite C (length units)
R91	z_A	component in the z direction of satellite A (length units)
R92	z_B	component in the z direction of satellite B (length units)
R93	z_C	component in the z direction of satellite C (length units)
R105	R	the final resulting radius of the circumcircle containing satellites A, B, and C on its circumference

The Input

The input to the program is shown as typed by the computer in **Figure C-2.**

```
----STP----
   SAT NOS....
              2
              4     1
             19
      DATE
           4.05     2
           1977
  LAN,NODE-SAT A
          30.00     3
          60.00
  SAT B
          60.00     4
         120.00
  SAT C
         285.00     5
          30.00
  IN.,DELT,&FINAL
  INCLINATION
          50.00
           1.00     6
          60.00
  ELAPSED NODE      7
          15.00
------------------------
  LONGS,LATS
          97.37
          47.73
         207.27
OUTPUT    32.80
         317.73
          32.80
  FOR INC.=......
          50.00
  RADIUS = ......
          51.95

          96.94
          48.65
         207.82
          33.33
         317.18
          33.33
  FOR INC.=......
          51.00
  RADIUS = ......
          51.24

          96.48
          49.57
         208.38
          33.86
         316.62
          33.86
  FOR INC.=......
          52.00
  RADIUS = ......
          50.53

          96.00
          50.48
         208.96
          34.38
         316.04
          34.38
  FOR INC.=......
          53.00
  RADIUS = ......
          49.83
```

Figure C-2 - Sample Input and Output of the R vs. Inclination Program. It gives the radius (R) of the circumcircle containing satellites 2, 4, and 19 on its circumference for varying inclinations. It also gives the correspong longitudes and latitudes. See graph on Figure 13.

```
            95.49              93.18
            51.39              55.00
           209.55             212.08
            34.89              36.85
           315.45             312.92
            34.89              36.85
FOR INC.=......    FOR INC.=......
            54.00              58.00
RADIUS = ......    RADIUS = ......
            49.14              46.40

            94.96              92.51
            52.30              55.89
           210.16             212.75
            35.40              37.31
           314.84             312.25
            35.40              37.31
FOR INC.=......    FOR INC.=......
            55.00              59.00
RADIUS = ......    RADIUS = ......
            48.45              45.73

            94.40              91.81
            53.21              56.77
           210.79             213.43
            35.89              37.76
           314.21             311.57
            35.89              37.76
FOR INC.=......    FOR INC.=......
            56.00              60.00
RADIUS = ......    RADIUS = ......
            47.76              45.06

            93.80
            54.11
           211.43
            36.37
           313.57
            36.37
FOR INC.=......
            57.00
RADIUS = ......
            47.08
```

Figure C-2 (continued)

Input #1 (2,4,19) - the numbers of the satellites comprising this spherical triangle as taken from Appendix B. (only for reference purposes).

Input #2 (4.05) - this is the month and the date of the run.

Input #3 (30,60) - the longitude of ascending node (Ω) and node angle (ω) of satellite A as found in Appendix B.

Input #4 (60,120) - the same as input #3 but for satellite B.

Input #5 (285,30) - the same as input #3 but for satellite C.

Input #6 (50,1,60) - these are the initial inclinations of the satellite orbits, the increment, and the final inclination.

Input #7 (15) - the elapsed node of all the satellites in their orbits.

The output

A sample output is listed above taken from the 24/24/14 constellation. The satellites involved are 2, 4, 19, 21 (9) (14) as listed in Figure 14. The three used for the program are 2, 4, 19. Only three of the four satellites are required to define the circle whose radius is being sought. One must be sure that the fact that there were four satellites equidistant from a given point is not coincidental only to the particular inclination at which it was analyzed. In other words it might be that satellites 2, 4, 19 and 21 do not all remain on the circumcircle as the inclination is varied. This was checked in the above example.

Essentially lines 10 through 13 compute the rectangular coordinates of the satellites from their Keplerian elements as was done in the Grader program.

Lines 14 through 18 compute the longitude and latitudes of each satellite at each inclination for reference.

Lines 20 through 22 compute the angular length of the sides which make up the spherical triangle--sides a, b, and c. This is done by taking the dot product of the corresponding satellite position vectors and then taking the arccosine to find the earth central angle between them. This angle is then the arc comprising one side of the spherical angle.

Lines 23 to 25 employ a trigonometric formula to give the angles of the spherical triangle once the three sides are done. The formula* states that:

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c} .$$

This in turn is also used to produce angles B and C.

Line 26 gives the radius of the circumcircle (R) which contains the three satellites in its circumference. This handy formula[4] is:

$$\tan R = \tan \frac{1}{2} \cdot \sec \frac{1}{2}(B + C - A).$$

* Taken from standard Math table.

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